Finding Zeros of Polynomials Algebraically

with help from the calculator

$$f(x) = 2x^4 - 3x^3 - 5x^2 - 6x - 18$$

Possible rational zeros: Factors of 18 divided by factors of 2

$\pm$ 18, $\pm$ 9, $\pm$ 6, $\pm$ 3, $\pm$ 2, $\pm$ 1 and

$\pm$ 18/2, $\pm$ 9/2, $\pm$ 6/2, $\pm$ 3/2, $\pm$ 2/2, $\pm$ 1/2 These reduce to $\pm$ 9/2, $\pm$ 3/2, $\pm$ 1/2

First suggestion, graph the function to see how many real roots (FIG 1). Since this is a 10 x 10 window, and largest (smallest) possible rational root is $\pm$18, redetermine appropriate window to include x = $\pm$ 18 (fig. 2)

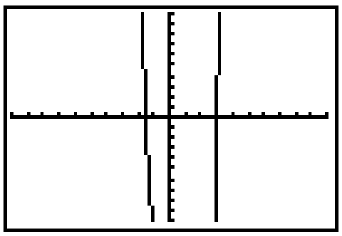

FIG 1

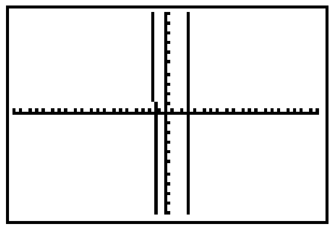

FIG 2

Since there are only two real roots, check them for rational.

Method one, check table for whole number zeros.

Notice that x = 3 is a zero (FIG 3). Since the other real zero crosses on the negative side, use the table ASK feature (FIG 4) to try the possible rational zeros close to the x-intercept, (or try a 2nd CALC 2 to see if it is rational.)

X	Y1	
-2	30	
-1	-12	
0	-18	
1	-30	
2	-42	
3	0	
4	198	

X=-2

FIG 3

X	Y1	
-.5	-15.75	
-1.5	0	
-4.5	1001.3	

X=

FIG 4

Notice from the table: -1.5 is also a zero Two factors of the polynomial are (x - 3) and (x + 1.5)

The degree of the polynomial is 4,leading to the conclusion that there are 4 zeros. Since two zeros are rational, we can use synthetic division to help find the other two.

First, perform synthetic division on the original equation using the diviso r= 3. Then take that quotient and use divisor = -1.5. The remainder will be zero and coefficients will be thoses of a quadratic.

Check your work. The first division yields 2, 3, 4, 6 = $2x^3 + 3x^2 + 4x + 6$. Now use these coefficients with divisor = -1.5. This yields 2, 0, 4 which means the other factor is $2x^2 + 4$.

The factors of f(x) are $(x - 3)(x + 1.5)(2x^2 + 4)$. Solve $2x^2 + 4 = 0$ for the other zeros.
Use thequadratic formula or any other appropriate method . x = $\pm i\sqrt{2}$

Therefore, the zeros of this polynomial are x = $\pm i\sqrt{2}$, -1.5, and 3